Linking 'The Book of Nature' and 'The Book of Science': Using Circular Motion as an Exemplar beyond the Textbook

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Linking 'The Book of Nature' and 'The Book of Science': Using Circular Motion as an *Exemplar* beyond the Textbook

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Abstract. Bacon exhorted the natural philosophers of his day to read and interpret the 'book of nature' by clever and cunning experimentation. The increasing scientific activity after Bacon and Galileo, however, quickly produced a second book. This was a book of interpretations of nature, namely the 'the book of science'. Newton went beyond Bacon and Galileo and developed an ongoing dialogue between these two books, a repeated give and take between mathematical construct and physical reality. Unfortunately, the physics textbook, the 'book of science' the students read, does not acquaint them with this style of reasoning. As an example of high-grade scientific thinking this paper discusses Newton's long struggle with the concepts of inertia and especially of 'centrifugal force'. In his quest to understand the dynamics of circular motion Newton clearly progressed through four levels of conceptualizations, leading to progressively less severe discrepancies, in his ascent to a full understanding of centripetal acceleration. While it is not possible or desirable to expect teachers or students to recapitulate high-grade scientific thinking, partial retelling of the intellectual struggle that was involved in establishing important scientific concepts must be seen as important. This kind of pedagogy, however, requires that physics teachers have a good understanding of the history of scientific ideas as well as the findings of cognitive science.

1. The 'Book of Nature' and 'The Book of Science'

On the eve of the Scientific Revolution, Sir Francis Bacon condemned the uncritical acceptance of Aristotelian physics and of scientific dogma in general. The failure of Aristotle's methods, he claimed, was due to the misreading of Aristotle. Rather than observe nature as Aristotle advocated, Bacon charged that Aristotelians studied only the deductive consequences of his first principles. This dogmatic theorizing, he argued, cut off the interrogation of nature (science) from its empirical base.

In his influential book *Novum Organum*, Bacon exhorted scientists (natural philosophers) to read and interpret the 'book of nature'. Bacon insisted that we must "put Nature to the rack" by clever and cunning experimenting. He believed that the secrets written in the 'book of nature' are accessible to us and that the phenomena we observe can be described and catalogued through imaginative classification. Moreover, he argued that regularity in nature can be captured by careful observation, all based on inductive reasoning. Bacon's methodological dictum "Truth emerges more readily from error than from confusion" (quoted in Kuhn 1967, p. 258) captured the new inductive spirit of investigation. His contemporary, Galileo, however, went further and suggested that in physics we must not only do clever experiments but also establish theories from which we can argue deductively to scientific facts by using mathematics. He believed that the 'book of nature' was written, not in the language of Aristotelian logic, but in the language of mathematics.

It is not surprising that the increasing scientific activity after Bacon and Galileo, quickly produced a second book. This was a book of interpretations of nature, consisting of theories, concepts and experimental confirmations, articles, proofs, critical discussions, confrontations, and so on, starting roughly at the time of Bacon and Galileo. We will call this book 'the book of science'. This book is now a "sprawling, rambling, collective work, with its own gradations as to depth and importance" (Eger 1993, p. 300).

By Newton's time, the notion that it is possible to go inductively from observations and experiments directly to laws and theories in a specifiable manner, became untenable. To illustrate the dialogue required between the 'book of nature' and 'the book of science' in the quest for understanding, we will take Newton's physics as the paradigmatic example of high-grade thinking in physics. This 'style' of doing physics has been consciously emulated by scientists in general since Newton's time. The Newtonian scientific style seems to have been an imaginative extension of the scientific methods of Bacon and Galileo.

It was clear to Newton that Bacon's ideas about how we should investigate nature were naive. It was probably also clear to him that Galileo's 'rational' physics was too idealistic. Newton combined in his approach Bacon's insistence that the proper way to interrogate nature was by clever experimentation with Galileo's model of mathematical analysis and deductive reasoning. The result was a dynamic and complicated conversation between the 'book of nature' and the 'book of science'. Newton's style was a "repeated give-and-take between mathematical construct and physical reality" (Cohen 1981, p. 177). Unfortunately, the physics textbook, the 'book of science' that students read, does not acquaint them with this style of scientific thinking.

2. The Science Textbook

In science (physics) teaching the textbook plays a dominant role and dictates both what is taught in science and how it is taught. The science educator Robert Yager, after examining science textbooks in the US, ironically stated that the most significant decision a science educator makes is the choice of a textbook. He went on to suggest that textbooks imprison science teachers in a belief that the instructional sequence of assign, recite, and test is guaranteed to produce knowledge. He went on to emphasize that direct experience is almost never offered, and laboratory work, if it occurs at all, is of the deductive-verification type. He claimed that high reliance on textbooks does not seem to produce scientifically and technologically literate people.

One of the shortcomings of textbooks is that they implicitly or explicitly promote what may be called a neo-Baconian approach to the reading of the 'book of nature', namely an empiricist-inductivist picture of science. This picture of science, essentially the one that Bacon described, seems to be still enshrined in many textbooks. It is the belief that laws and discoveries are a guaranteed consequence of systematic observation that is based on a specifiable scientific method. Another shortcoming of many textbooks is the implication that a clearly (to the instructor) presented lesson is guaranteed to produce knowledge in the student.

Historians and philosophers of science, however, tell us that scientific concepts and theories do not follow from observation in a simple inductivist manner and that scientists study a world of which they are a part, and not a world from which they are apart.

Students, therefore, are frequently indoctrinated into an unsceptical acceptance of an inductivist-empiricist picture of science. Moreover, learning is seen as a slow accumulation of knowledge through practice, where the learner is assumed to be, in the John Locke tradition, a tabula rasa. We must remember that science (physics) teachers learned their science from textbooks, then teach from textbooks that largely emphasize memorization of scientific facts and the recitation of algorithms in an ongoing rhetoric of information.

3. Kuhn's Normal Science and The Physics Textbook

Thomas Kuhn, probably the most influential contemporary philosopher and historian of science, was also a trained physicist. His ideas about the role of textbooks in physics are therefore important for physics educators. Kuhn maintained that students of physics learn physics by studying specific applications and concrete examples – what he calls *exemplars* – or "the concrete problem-solutions that students encounter from the start of their scientific education, whether in laboratories, on examinations, or at the end of chapters in science texts" (Kuhn 1962, p. 187). Kuhn wondered why we almost never find in textbooks a description of the sort of problems that the professional may be asked to solve. Clearly, Kuhn implied that 'the book of science' is almost never read by students of physics, even at the university level.

Kuhn believed that *exemplars* are at the heart of the education of both the student of elementary physics and the mature scientist working within the confines of 'normal science'. According to Kuhn, textbooks are *pedagogic vehicles for the perpetuation of* 'normal science'. Exemplars are model solutions of what scientists consider an important class of problems that textbooks excel in demonstrating. What distinguishes science teaching from teaching in the humanities, Kuhn remarked, is precisely the almost exclusive reliance on textbooks.

Though it is generally true that different science textbooks in a given science display different subject matter Kuhn believed that they do not differ in substance and conceptual structure. In the humanities and in many social sciences, on the other hand, textbooks differ fundamentally in the way they "exemplify different approaches to a single problem field". It is interesting to note that in the mature sciences "there is no apparent function for the equivalent of an art museum or a library of classics. Scientists know when books, and even journals, are out of date" (Kuhn 1987, p. 256). When science changes, textbooks are rewritten to accommodate this change.

Kuhn, clearly, and without apology, recognized the dogmatic nature of textbook-centered science education:

> ... Though scientific development is particularly productive of consequential novelties, *scientific education remains a relatively dogmatic initiation into a pre-established problem-solving tradition that the student is neither invited nor equipped to evaluate* (italics mine) (Kuhn 1962, p. 000).

For Kuhn then contact with the 'book of nature' means being engaged in solving problems based on the commonly recognized *exemplars* of a science (physics). It should be stressed, however, that Kuhn did not see this activity as the kind of algorithmic problem-solving that seems to be at the heart of conventional science (physics) teaching. Rather, he argued, that "by doing problems the student learns consequential things about nature". In elementary physics, for example, these are the problems that are related to such exemplars as the *inclined plane*, *billiard ball collisions*, *the conical pendulum*, *Atwood's Machine*, *ripple tank experiments*, and the *electronic air-table*. The physics imbedded in problems that are part of an exemplar, Kuhn insists, should be developed and sequenced so that the laws (for example Newton's second law, $F = ma$) are not seen as a finished product of mathematical formulation, to be committed to memory and then applied to problems algorithmically. Rather, Kuhn argues that students should learn to think of such laws as 'symbolic generalizations' that *gain new meanings in different contexts.*

Unfortunately, Kuhn's argument that reading the 'book of nature', or "doing problems is learning consequential things about nature", may be true only in a restricted sense. Physics teachers tend to trivialize such recommendation by providing mostly inconsequential problems. For the majority of students, *doing problems is to memorize 'scientific facts' and practice algorithms.*

Kuhn agreed that textbook-centered science teaching has been very successful in producing proficient scientists for research and technology. Nevertheless, he had misgivings about its effectiveness in producing the kind of high-grade thinking required to periodically examine the foundations of a science:

> ... But for normal scientific work, for puzzle-solving within the tradition that the textbook defines, the scientist is almost perfectly equipped ... Even though normal crises are probably reflected in less rigid educational practice,

> *scientific training is not well designed to produce the man who will easily discover a fresh approach* (italics mine). (Kuhn 1962, p. 166)

The textbook in science (physics) then is a book about 'the book of science'. Textbooks are logical reconstructions of science and dictate the form and presentation of science. The format of physics textbooks and presentation is highly stylized and the variation across textbooks is insignificant.

Students are seldom asked (or able to) read the 'book of science', they encounter science (physics) by reading textbooks. Understanding science then comes from reading the results that have been distilled and enshrined, 'verified' for all times, and presented as a 'finished product'.

The science textbook will probably be with us for some time to come. It may even be necessary for the education of the scientist, as Kuhn thought. Textbook-centered teaching, however, must be revised to produce a greater number of creative scientists and scientifically and technologically literate people. To achieve this, science educators must recognize and understand the dynamic dialogue between the 'book of nature' and the 'book of science' mediated by creative scientists. In other words, students should be aware of this exciting creative act and learn how to read 'the book of science', and not just memorize passages from textbooks.

4. Prescientific Ideas of Students

Students, of course, have 'read' and interpreted 'the book of nature' since their childhood. Children seem to have commonsense understanding of the world that is based on kinesthetic memory and conceptions about motion, heat, electricity and biology. Children learn about the world through experience and the use of language, the wealth of visual and verbal information they receive from TV and books. Children then generate their own 'theories' as to how and why things behave as they do.

Much research has been done in identifying these 'preconceptions' and classifying them for the major age groups in such domains as motion and electricity. Roger Osborne (1984) found that young students understand motion around them, using a sequence of 'mini theories' he labelled 'gut dynamics', 'lay dynamics' and 'physicist's dynamics'. Gut dynamics is intuitive, spontaneous, is largely nonverbal, and allows children to cope with common occurrences around them. Examples of gut dynamics includes, "heavy things fall faster" and "things need a push to get them going". Lay dynamics is based on form and content of the language the child grows up to speak and the images conveyed by those she is in contact with and the media and the books she reads. Examples of lay dynamics would be the idea that "astronauts are weightless in the space shuttle" and "if there is no force there is no motion".

Physicist's dynamics is the counterintuitive world of physics texts, experiments and problems students solve in class. Osborne found that students were able to learn 'physicist's dynamics' which enabled them to operate in the idealized world of the physics laboratory and the examination paper. Unfortunately, many did not develop an integrated and coherent view of dynamics and retained a mixture of gut and lay physics. As Osborne graphically puts it:

> Gut dynamics enables one to play hockey, lay dynamics one to talk about Star Wars, while physicist's dynamics enables one to do physics assignments. There is no problem! (Osborne 1984, p. 506)

What is worrying to physics educators is that a high percentage of students, even though they can solve fairly sophisticated physics problems, still operate with gut and lay physics ideas in everyday life.

A plethora of informative and well argued papers on the topic of dynamics (force) has appeared recently in science and physics education journals (Feingold and Gorsky 1991; Gunstone and Watts 1985; McCloskey 1986; Sadanand and Kess 1990, 1992; Terry and Jones 1986). The main findings in such articles are well summarized in a comprehensive paper in the *Physics Teacher* by David Hestenes et al. (1992). These are:

1. Common sense beliefs about motion and force are generally incompatible with Newtonian concepts,
2. Conventional physics instruction produces little change in these beliefs, and
3. This result is independent of the instructor as well as the mode of instruction.

The most surprising finding is that misconceptions seem to spontaneously disappear for those who study physics as a major. What is especially interesting is "the paradoxical fact that few physicists can recall having believed, let alone having overcome, any of the misconceptions" (Hestenes et al. 1992, p. 151). The authors insist, however, that research has established unequivocally that everyone has such misconceptions before learning physics, even the great physicists. In a recent article Steinberg et al. (1990) show how even "Newton's progress was blocked by a web of misconceptions". Newton clearly had an especially difficult time in giving up the idea of centrifugal force and seems to have rejected the principle of inertia for a long time.

Physics teachers, too, forget the conceptual struggle they had in achieving an expert understanding of the notion of force in Newtonian physics. They teach lessons of exemplary clarity (to them) and believe that therefore it will also be clear to the students.

The international science education community seems to have concluded that the findings listed above have serious implications for the teaching of physics. These are:

1. Common sense beliefs that students have should be regarded, not as misconceptions, but as reasonable hypotheses grounded in everyday life.
2. Students should be encouraged to articulate these hypotheses which are generally based on personal kinesthetic memory.
3. Physics teachers should make it as their priority to identify these hypotheses. This act should be regarded as a necessary but not sufficient prerequisite to successful physics teaching.

4. Physics teachers should try to overcome misconceptions by offering the coherent conceptual system of Newtonian physics in interesting and new ways.

It seems that the most common misconceptions (pre-Newtonian hypotheses) are connected with the *impetus* concept of motion, often referred to as *transfer of force*; with the force involved in circular motion, often referred to as *centrifugal force*; and with interaction forces in Newton's third law, often referred to as the *conflict concept of interaction.* Hestenes et al. admit that conventional textbook-centered teaching does seem to produce a Newtonian understanding of these concepts for a chosen few. They argue, however, that even for these students this is an inefficient route. Other instructional techniques, such as contextual teaching (Stinner 1989, 1993), group discussions involving discrepant events, conceptual bridging (Driver 1989), and skills in diagrammatical representation of forces (Hestenes et al. 1992) must be used *prior* to using algorithms in solving problems. Hestenes et al. suggest that this is probably best accomplished by teaching the Newtonian unitary concept of force *before* the traditional problem-solving activity commences.

The conceptualization that guides children's understanding and judgements about the world around them is partly based on their own direct interpretation of 'the book of nature'. It is also based on 'indirect' sources, such as their reading of popular science, watching TV and learning their 'school science'. For the scientist, however, an understanding of the world around us, is the result of an ongoing dialogue between 'the book of nature' and 'the book of science'.

5. The Dialogue between 'The Book of Nature' and 'The Book of Science'

According to Aristotle, to think scientifically we must not only be able to recognize something as a scientific fact but also know why it is a scientific fact. Statements such as 'the earth revolves around the sun' or 'ultraviolet rays cause cancer' are recognized as scientific facts by most students, but few of them would be able to give good reasons for believing what those statements claim. To be able to do so requires acquaintance with 'the book of science'. Acquaintance with the 'book of science', in turn, implies understanding a network of definitions and concepts, connected to laws and principles, imbedded in scientific theories.

This kind of understanding, however, comes slowly as the result of conceptual development through many levels of understanding. Piaget investigated the question of how human thought is capable of producing scientific knowledge. Specifically, he asked the question: "How is it that the human mind goes from a state of less sufficient knowledge to one judged more sufficient by experts in a particular area of science?" The answer he gives is his principle of equilibration, or self-regulation (Rowell 1989, p. 141). The mechanism of equilibration , very simply put, comes into play when we are at a loss to explain a phenomenon or an aspect of a phenomenon using our existing conceptual apparatus. This inability to explain produces a mental discomfort (cognitive disequilibrium) that demands a response. The response consists of conceptual readjustments in a multi-step process involving feedback loops - feedback from the effect of an action provokes a reassessment of a situation and that, in turn, results "in a continuation of the action in modified form, which is followed by feedback ... and so on" (Rowell 1989, p. 143).

Piaget goes on to describe a progressive sequence of levels, or "phases of compensatory constructions" as a consequence of a perceived mismatch between anticipation and observation. In other words, at each stage there is progressively less mismatch between the facts given by nature and the application of the individual and her knowledge framework. In the first stage there is a conservative response to the mismatch, a general resistance to change. In the second stage there is progressive theory change (accommodation), retaining much of the original theory but integrating the disturbance as a new variation. Finally, in the last stage, the reorganization begun in the second stage is completed: the new theory now accommodates the disturbance. The mental discomfort disappears and the new theory is 'symmetrical', that is the initial disturbance is now anticipated and not eliminated. These stages shade into each other and are never clearly delineated. The dialogue between 'the book of nature' and 'the book of science' is completed.

Piaget argued that this dynamic process of equilibration by way of a progressive sequence of levels describes the conceptual change of both the practising scientist and the student learning science.

Research strongly suggests that there are clear parallels between students' intuitive conceptions in physics (kinematics, mechanics, electricity, heat) and historical prescientific conceptions (McKloskey 1983). These findings suggest that it may be desirable to recapitulate the historical process in our physics classes. Closer examination of the complex thinking involved in such scientific discoveries and conceptualizations as inertia and circular motion, however, shows that the quest for achieving a full recapitulation is unreasonable and probably undesirable. It is unreasonable to expect students to completely recapitulate the high-grade thinking of a Newton and it is undesirable because it would be a time-consuming enterprise to present the full historical picture.

A plausible case, however, can be made for a limited case of recapitulation of the historical process in domains, such as pre-Newtonian mechanics, early heat theory and electricity, that are experientially familiar to students (Stinner 1994).

We will look at Newton's struggle to understand circular motion. This topic or concept is generally presented in the time-frame of 1–2 classes that may involve simple demonstrations, culminating in the derivation of the equation $a_c = v^2/r$ and the solving of problems that are suited to this 'exemplar' of elementary physics. There is seldom any reference to a conceptual struggle and certainly very little historical context presented. First, I am claiming that presenting an important topic or concept as a limited historical case study is superior to the conventional one. This approach shows the student that even a great scientist like Newton had difficulty in understanding such concepts as inertia and centripetal acceleration and it also allows students to become acquainted with the history of physics. Admittedly, Newton developed these key concepts from an incomplete background knowledge available, and therefore was the first to add these to the 'book of science'. Secondly, I believe most major topics and concepts, or Kuhnian exemplars in elementary physics, mentioned above, can be presented this way. Finally, I am convinced that this approach will produce better scientists and certainly more scientifically literate students who do not become scientists.

6. Physics Beyond the Textbook: Linking 'The Book of Nature' and 'The Book of Science'

Newton struggled with the concepts of inertia and centrifugal force for many years. He went through several stages or "levels of understanding" in trying to link the two books, that finally allowed him to consistently place these concepts in his emerging dynamics, as well as establish the relationship between them. These "levels of understanding" can be seen as a historical example of a Piagetian conceptual struggle scientists (and students!) must experience in order to deal with discrepant events that conflict with the their existing conceptualizations. In Newton's case that was a knowledge base originally inherited partly from the late medieval natural philosophers and also from Galileo. Each level produced a discrepant situation that demanded a strategy for conceptual change. Newton seems to have required several such levels to free himself from the medieval idea of impetus and transform this notion into the modern concept of inertial mass. Similarly, in his quest to understand the dynamics of circular motion we can identify four levels of conceptualizations, leading to progressively less severe discrepancies, in his ascent to a full understanding of centripetal acceleration (Stinner 1994).

Textbook accounts of discoveries and conceptualizations in physics, such as we find in Newton's dynamics, get around the challenging problem of giving a historical discussion of how such basic concepts as inertia and centripetal acceleration were formulated. This is often achieved by simply presenting them as if they seemed self-evident and came full-blown to the mind of the great man, shortly after the apple fell on his head. Similarly, when presenting the physics of optics, heat, electricity and magnetism, we must try to present the development of major concepts as a struggle to understand on the part of the scientists.

Piaget argued that the science (physics) student who struggles to understand difficult concepts such as inertia or centripetal acceleration, seems to experience similar 'levels of understanding' that could be analyzed using his idea of mental equilibration and progressive levels of attaining it. Recently I have tried to summarize for physics educators the conceptual development of the notion of force from Aristotle to Einstein (Stinner 1994, 1995). This is a science story, consisting of many separate historical case studies connected by the common theme of "force". I suggested appropriate analogies, limiting case analyses, thought experiments and imagistic representations toward a partial recapitulation of the historical process of the concept of force. Here I will present only a limited recapitulation of Newton's struggles to understand circular motion and how it was connected to inertia in his general dynamics.

Mark Silverman, a Harvard physics professor, interested in presenting his science in more authentic ways, recommends that we teach science (physics) by exposing the humanistic ties that link science to the general intellectual heritage of the student:

> . . . I have found that most students and many teachers regard science courses, and especially textbooks, as providing material the truth of which has been established beyond doubt . . . most nonscientists (including teachers) unfamiliar with the nature of science, envision the laws of nature as simply 'being out there', ready for picking like fruit from a tree. Textbooks, too, often foster that image; on page after page follow the experiments, the data, the equations, the conclusions – all neatly ordered, seemingly unconfused, devoid of struggle and human emotion. But this is not how science works. A science educator must disabuse his classes of these misconceptions, to show that there is in the creation of science, as in the creation of great art and great music, the drama of human enterprise. (Silverman, 1989, p. 52)

The following is an example of how the major ideas or concepts in physics can be made more authentic, along the lines recommended by Silverman. I have chosen the physics of circular motion, a difficult topic in elementary physics and generally taught by way of an algorithmic approach. without giving any historical background. The conventional approach encourages memorization and recitation, and often leads to an incomplete conceptual understanding.

7. Circular Motion: An Examplar Placed in History

7.1. PRE-NEWTONIAN ATTEMPTS TO UNDERSTAND *CIRCULAR MOTION*

Aristotle thought that the natural motion of a celestial body, composed of the fifth element "aether", was circular. Referring to the state of affairs in the physics of motion in the early seventeenth century, Westfall says:

> Circular motion, which appeared so natural in the context of the Aristotelian world view as to be the symbol of perfection, became an enigma in the mechanical universe. Until the riddle was solved, a workable dynamics was impossible (Westfall 1971, p. 19).

Another constraining factor to the modern description of circular motion of, those who came after Galileo, was the continued use of the same geometric methods and geometric ratios that he used in his kinematics. Galileo seemed to have embraced the idea of natural circular motion as a state of equilibrium between whatever caused bodies to turn them into a circular path and the force they exert against it (Westfall 1971, p. 82).

From everyday experience it was well known that a potter’s wheel would fling off bits of clay. This was the Aristotelians’ objection to the rotation of the earth. According to Galileo centrifugal force could never remove something from a circular motion, if there is an opposing force toward the center, however weak (see Figure 2). Galileo, of course, was wrong. Galileo’s geometric and kinematics approach to understanding the dynamic concept of centrifugal force was inadequate to solve this problem.

Investigators in the seventeenth century, therefore, believed that a body exerted a *centrifugal force* when rotating, a term coined by Huygens, meaning a “center-fleeing” force. Westfall argues that Descartes influential treatment of circular motion was largely responsible for perpetuating the equilibrium picture: Descartes believed that the tangential motion of a particle moving in a circle as the resultant of a circular motion and a radial tendency away from the center. Huygens also believed that weight and centrifugal force were complementary phenomena. He believed, like Descartes, that weight is caused by deficiency of centrifugal force. To find the “law of centrifugal force” he first showed, by careful geometric analysis, that centrifugal force increases in proportion to the velocity squared and decreases in proportion to the diameter of the circle increases.

Circular motion was one of the central riddles that the seventeenth century confronted, because “... a considerable effort of the imagination is required to see as uniformly accelerated a motion that is constant in speed” (Westfall 1971, p. 174). Huygens understood as early as in 1665 that acceleration of an object travelling with a constant speed v in a circle of radius R was given by v^2/R. He first showed that if a body moves in a given circle with a velocity equal to that which it would acquire in falling from rest through half of the radius of the circle, the ‘centrifugal’ force exactly equals its weight. This could be demonstrated in a conical pendulum where the ‘centrifugal’ force overcomes gravity. When the cord made an angle of 45° with the vertical, it was clear that the centrifugal force was equal to the weight of the bob (see Figure 1). Huygens’ experiment with the conical pendulum at 45° established a standard unit for centripetal force and lead to the relationship between centrifugal force (as Huygens still referred to it) that we today write as $F = m\, v^2/r$.

Huygens quickly applied his knowledge of circular motion and force to the problem set by Copernican astronomy. The question he tried to answer was one that Galileo was unable to deal with: “If the earth turns on its axis, why are bodies not thrown off its axis”? Galileo discussed this problem earlier without a good understanding of circular motion (see Figure 2).

In 1667 Huygens first tried to compare the centrifugal acceleration calculated from the size of the earth and its speed of rotation with the measured acceleration due to gravity. Unfortunately, both the accepted radius of the earth was inaccurate and the value of g, measured directly by using falling bodies was even less accurate. He improved the determination of g by using his newly found formula for the period of the pendulum. A fully satisfactory comparison, however, had to wait for

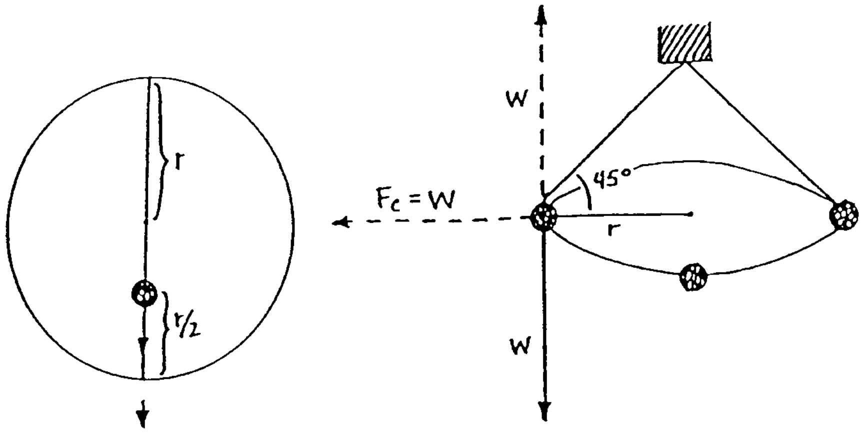


Figure 1. Huygens identified the 'centrifugal' acceleration with weight. He first showed that if a body moves in a given circle equal to that which it would acquire in falling from rest through half of the radius of the circle, the 'centrifugal' force exactly equals its weight.

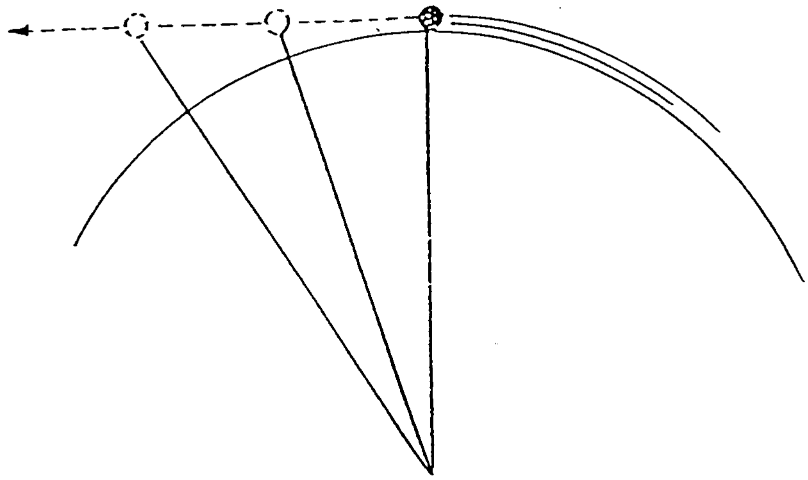


Figure 2. Huygens calculated the radius of the sphere on which a given body would have a centrifugal force equal to its weight when the sphere turns at the earth's rate of rotation. Huygens knew that for a given angular velocity the centrifugal force varies directly with the radius. He found that the earth would have to be 265 times larger than it is, the acceleration of gravity remaining unchanged.

about a decade later when Picard established the diameter of the Earth as 57060 toises, or 12554 km, very close to today's accepted value of 12756 km. Picard's value was used by Newton in his later work (Westfall 1991, p.191).

Galileo showed many years earlier that the period of a pendulum is proportional to the square root of the length. But his relationship between period and the length of the pendulum was based on purely empirical investigation and yielded only a proportionality statement. Huygens derived the relationship between the period

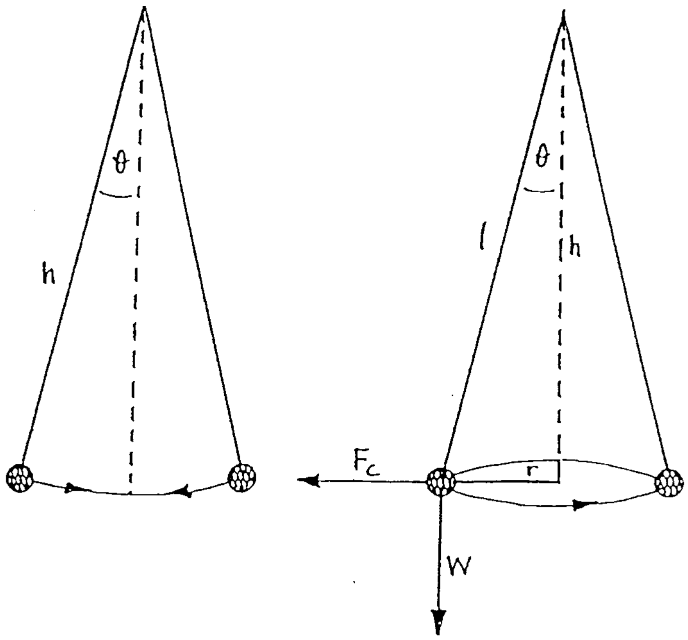


Figure 3. Huygens derived the relationship between the period and the length (for small angles), showing that the period of a conical pendulum, for small angles, is approximately the same as the period of a simple pendulum. He already knew that $a_c = v^2/r$. He could now show that $T = 2\pi(l/g)^{1/2}$. This equation allowed him to find the value of $g$to an accuracy of three significant figures. He travelled south to the equator to make accurate measurements of the value of g as a function of latitude.

and the length (for small angles), showing that the period of a conical pendulum (for small angles) is approximately the same as the period of a simple pendulum. This way he was able to express the value of g in terms of length and period (see Figure 3).

We will now discuss how Newton tried to understand circular motion. He reasoned much along the lines of Huygens', but was able to go beyond Huygens' physics and make connections that lead to the complete understanding of orbital motion of the moon and the planets.

7.2. NEWTON'S STRUGGLE WITH *CIRCULAR MOTION*

In one of his early attempts to quantify circular motion Newton reasoned that revolution through half a circle is equivalent to a perfectly elastic rebound, which requires a force great enough, first to stop a body's forward motion and then to generate an equal motion in the opposite direction. Westfall points out that the analogy is misleading and that "the difference from impact is that a body in revolution never stops and that the endeavour away from the center is exerted uniformly over a period of time unlike the impact which produces a change of motion" (Westfall

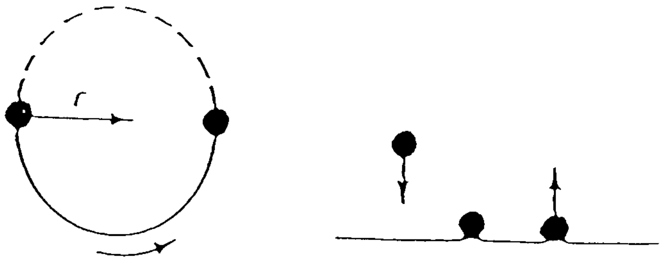


Figure 4. In his first attempt to understand to quantify circular motion Newton reasoned that revolution through half a circle is equivalent to a perfectly elastic rebound, which requires a force great enough, first to stop the body's forward motion and then to generate an equal motion in the opposite direction. However, he found that this analogy does not hold and led Newton to dimensionally incommensurable results.

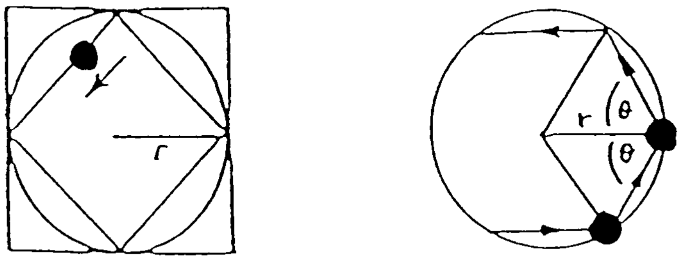


Figure 5. Newton next imagined a square to be circumscribed around the circle and the ball to follow a path inside it. Taking the component of motion perpendicular to the side, he set down an expression which compared the force of one impact, in which the component is reversed, to the force of the ball's motion. (See Arons (1992) for a modern version of Newton's calculations.)

1971, p. 351). This analogy led Newton initially astray because "the force from the center is dimensionally incommensurable with the force (impulse) exerted in impact" (p. 351) (see Figure 4).

In another attempt to quantify the force in circular motion Newton imagined a square to be circumscribed around the circle and the ball to follow a path inside it. Taking the component of motion perpendicular to the side, he set down an expression which compared the force of one impact, in which that component is reversed, to the force of the balls motion (see Figure 5). Newton subsequently realized that "if the number of sides of the inscribed and circumscribed polygons is increased, the ratio of force for one circuit continues to equal the ratio of the length of the path to the radius" (Westfall 1971, p. 354). This approach yields the correct result $a = v^2/r$.

In his third attempt Newton argued that the 'centrifugal' force of a revolving body is such that an equal force, applied to a body of equal mass during the time that the body revolved through one radian, would generate an equal linear velocity in the other body and move it from rest through half the length of a radian (see

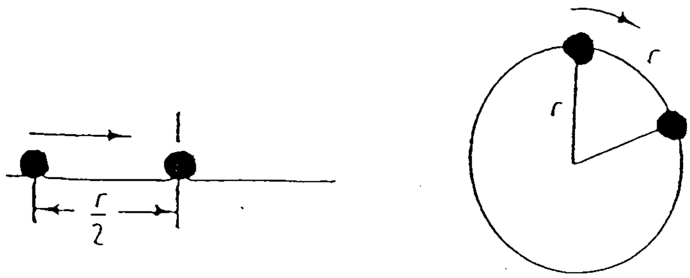


Figure 6. In his third attempt Newton argued that the centrifugal force of a revolving body is such that an equal force, applied to a body of equal mass during the time that the body revolved through one radian, would generate an equal linear velocity in the other body and move it from rest through half the length of the radian. A simple calculation shows that this approach yields the correct expression for centrifugal force, namely: $F = m\, v^2/r$.

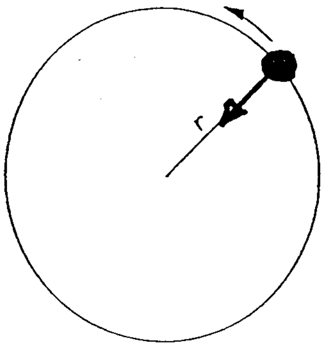


Figure 7. Until Newton was able to think of the force involved in circular motion as “center-seeking”, rather than “center-fleeing”, his dynamics could not be applied to the motion of the moon and the planets.

Figure 6). This approach also yields the correct result that $a = v^2/r$. Although the last two attempts gave the correct magnitude of the force it did not suggest the correct direction. However, until he was able to think of the force as “center-seeking”, rather than “center-fleeing”, his dynamics could not be applied to the motion of the moon and the planets (see Figure 7).

Newton next checked Galileo’s figure for the acceleration of gravity. Using a conical pendulum 81 inches long, and reasoning along the lines of Huygens, he showed that the value of g is about 400 in/s^2. This is about 10 m/s^2, very close to the value we accept today in textbooks (see Figure 8). Westfall writes the following about Newton’s reasoning:

> In some ways not disclosed, he convinced himself that he had measured a number of swings with the thread inclined at 45°. A series of ratios led to the figure of 1512 ticks in an hour. In roughly three-eighth of a second, the bob of the pendulum travels the length of the radius of its circle, and in the same time

> a body falling from rest would travel half as far, or about 50 inches in half a second and therefore 200 inches in one second, an excellent approximation to our value.
>
> . . . Throughout the calculations Newton employed results that set distance travelled from rest under uniform acceleration proportional to the square of the time. Hence his references to the "force of gravity" have the implicit effect of translating Galileo's kinematic to dynamics (Westfall 1971, p. 357).

It is not clear when Newton made the switch from center-fleeing to center-seeking nature of the force when a mass is in circular rotation. It was probably at this time when he considered the conical pendulum from the point of view of the person rotating the mass.

Finally, he managed to derive the formula for "centrifugal" force in a more economical and elegant way. Here he used the results of Galileo's kinematics of free fall and applied them to the dynamics of a revolving object (see Figure 9).

Newton was now ready to apply his conceptual apparatus consisting of full understanding of centripetal acceleration and inertia to calculate the period of the moon (see Figure 10).

Reading historical accounts such as Westfall's and Cohen's discussion of Newton's struggle to understand circular motion and how it fitted into his emerging dynamics one gets a distinct impression of Newton having reached the level of a mental equilibrium by about 1680. As late as 1679 Newton still "appears to have viewed circular motion as an equilibrium of opposing forces" (Westfall 1971, p. 427). A few years later, however, the idea of centripetal force was enshrined in the Principia (Book I, Proposition 4), as clearly explained in the recent *Newton's Principia: The Central Argument* (Densmore 1995).

8. Classroom Representation

Using a historical approach along the lines suggested here requires a good understanding of Newtonian dynamics and more than cursory acquaintance with the historical context on the part of the instructor. The historical approach may be more time consuming than the conventional textbook-based teaching but it has several clear advantages. First, the historical approach is generally more interesting and motivating, secondly, it engages the student in a conceptual development of Newtonian dynamics that is 'parallel' to the original conceptual struggle, and thirdly, it makes connections to other important and exciting aspects of nature than in the conventional textbook-centered approach is either omitted or mentioned only in a contrived fashion. In other words, presenting an exemplar this way would be a response to Kuhn's injunction that problems embedded in exemplars should be presented to (1) help a students 'learn consequential things about nature' and (2) to show students that important concepts like circular motion must be understood as 'symbolic generalizations that gain new meaning in different contexts'. Conceptual development then should be seen in terms of linking the 'book of nature' and

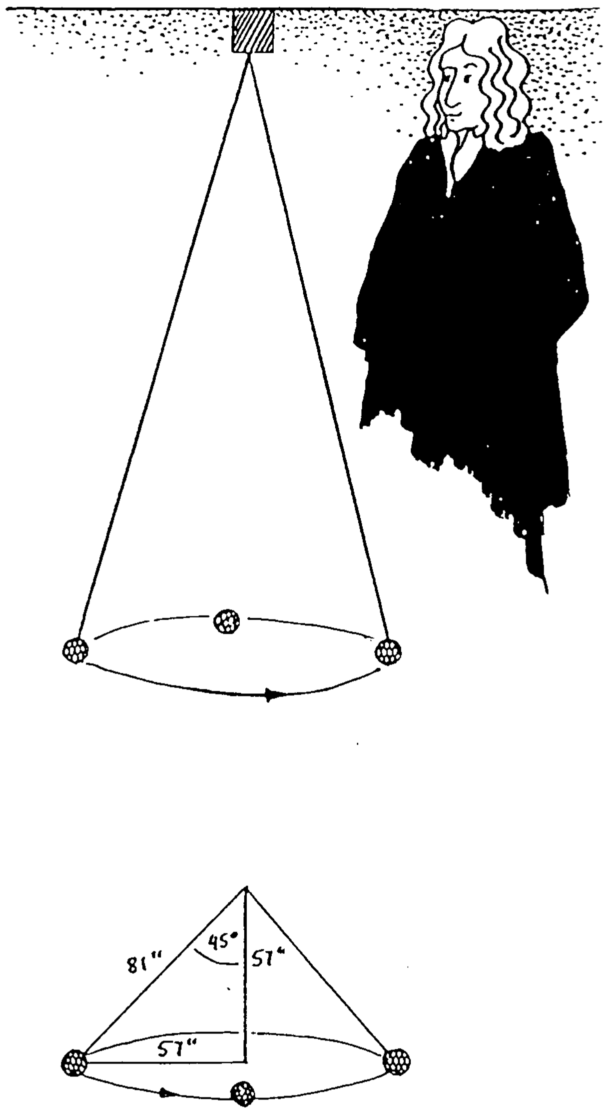
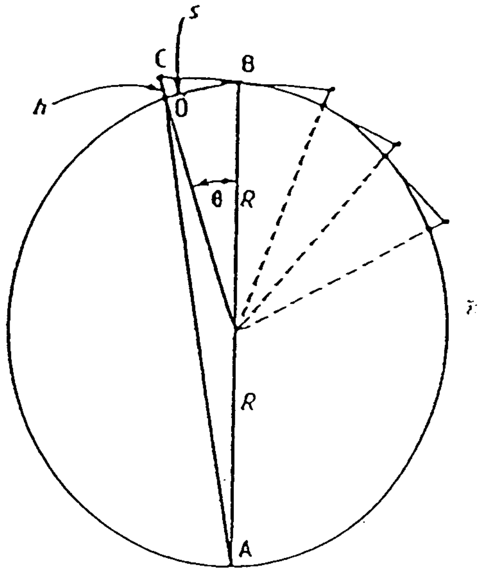


Figure 8. Newton checked Galileo's figure for the acceleration of gravity. Using a conical pendulum 81 in long, and reasoning along the lines of Huygens, he showed that the value of g is about 400 in/sec^2 squared, a value close to the present accepted value of about 10 m/s^2.

the 'book of science' by way of *progressively higher levels of sophistication in increasingly richer contexts of involvement.*

The discussion of circular motion could start with the experiments and arguments of Huygens, as suggested above, followed by a presentation of Newton's struggle with this important concept in physics. Each of the levels of this struggle can be presented historically and then engage the student in discussion, experimentation and problem solving.



$\Delta ABC = \Delta DCB$ for small θ

$s = vt$

Let $h \simeq \frac{1}{2} a_c t^2$

$\Rightarrow \frac{h}{s} = \frac{s}{2R}$

Substitution gives $a_c = v^2/R$

Figure 9. This conceptualization of centripetal acceleration in terms of free fall around the earth was still used in textbooks in the 1950s.

The first level is clearly a 'naive' attempt to understand the force involved in rotational motion and its discussion usually generates much interest. The second level (Figure 2), for example is nicely discussed by Arnold Arons in his book, *A Guide to Introductory Physics Teaching* (Arons 1992, p. 139). The third level is actually a discussion of the motion of a conical pendulum and is an attempt to reconcile linear acceleration with acceleration. The final attempt is one that physics teachers will recognize as a precursor to the textbook version students will first encounter.

At this point a conventional textbook version of centripetal acceleration can be given, for example the PSSC version. There are of course, many different approaches to deriving the famous formula and can be found in such journals as *The Physics Teacher* and *Physics Education.*

The instructor who wants to enrich the development of this important topic could add the following exercise as enrichment could proceed as follows:

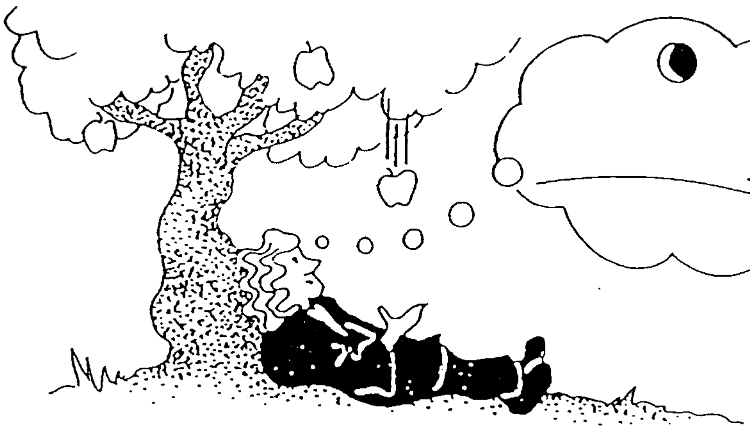


Figure 10. Using his inverse square law of universal gravitation attraction and his new understanding of centripetal force, Newton tested this mathematical model on the moon's orbit. Newton calculated that the moon moves as if it were attracted to the earth with a force that is 1/3600 of the strength of the gravitational force with which the earth pulls an objects at the surface. Newton knew that the distance to the moon was about 60 earth radii. He also assumed that the gravitational attraction between an object and the earth diminishes inversely as the square of the distance. Moreover, he assumed that the gravitational attraction of the moon on the earth is equal in magnitude and opposite in direction to the gravitational attraction of the earth on the moon (his third law). Finally, by now he also understood that centripetal acceleration was provided by the earth's gravitational attraction on the moon.

1. Present a geometric argument to find the 'average' acceleration of an object moving in a circle of radius r with a constant speed v, using vector addition and the cosine law (see Figure 11a.). First, students can use a table calculating the average acceleration as the angle gets smaller and smaller (using unit radius and unit time for the period makes these calculations easier). Secondly, the expression obtained for centripetal acceleration students could try to find the limit of the expression as the angle θ approaches zero. This is a good place to connect the physics of circular motion with the newly developed calculus by Newton and others. For example, L'Hospital, a contemporary of Newton, developed very useful rule for finding limits for expressions of the type discussed in this example (see Figure 11b).

2. Connect to everyday experiences: Compare the forces on a car tire, when the car is moving at 100 km/h; the car's wheels are rotating freely when the car is on a platform in a garage; the angular speed of a super centrifuge that produces $100g$ effect on the rim; the coefficient of friction required to hold a car in curve for a given speed; the period of rotation of a circular space station that produces an 'artificial' gravity of one g; and so on. At this point students would be ready to tackle the myriad of problems that textbooks present with more enthusiasm and understanding than they would have had, going the conventional,

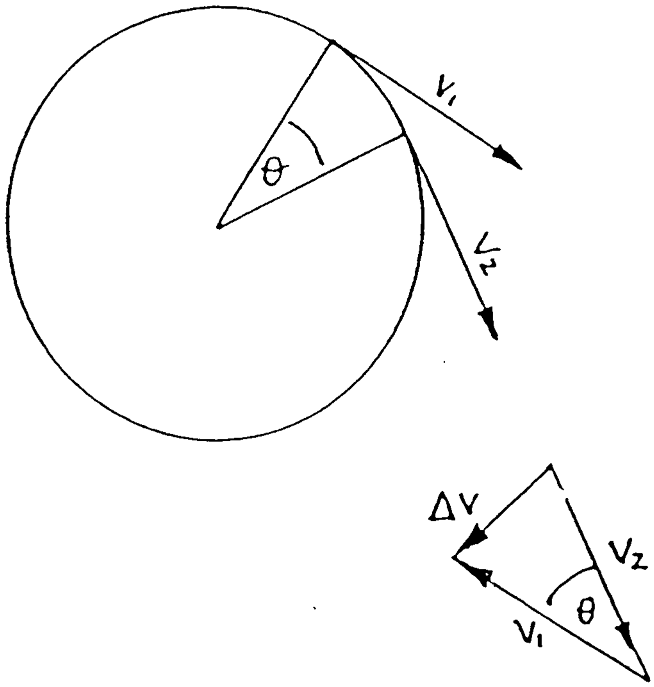


Figure 11. The following is a pre-calculus treatment of the centripetal acceleration of an object moving in a circle of radius r at a constant speed of v. The average acceleration is given by $a = \Delta v/\Delta t$ and the $\Delta v = v_2 - v_1$ using the cosine law $\Delta v = (v^2 + v^2 - 2vv\cos\theta)^{1/2}$, since v is the magnitude of both vectors v_1 and v_2, but $t = \theta(/2\pi)T$, where $T = 2\pi\ r/v$. It follows then that $a = (v^2/r)\{2(1 - \cos\theta)/\theta^2\}^{1/2}$. (a) Students can now substitute values for θ, from $\pi/2$ to about $\pi/60$, for example, and find out what value is approached. (b) Students may want to use their knowledge of limits and evaluate Lim. as $\theta \to 0$ of $(1 - \cos\theta)/\theta^2$. Clearly, the answer must be 1/2. The limit can also be evaluated by applying L'Hospital's rule that was first used about the time Newton was working on his *Principia*.

textbooks-centered route. Textbooks are now used as important reference only and not for providing guidance for conceptual development.

Finally, it should be emphasized that only the instructor is expected to follow the details and the mathematics of the arguments presented in this paper. It is not necessary nor desirable to recapitulate the high-grade scientific thinking of a Huygens or a Newton. But partial recapitulation is possible that could be exciting for the instructor and enriching for the student. It is arguable that the conceptual and mathematical details of the various stages may be as difficult as the concept of circular motion itself, but not if it is filtered through the agency of an experienced and enthusiastic instructor.

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